

**University of Mumbai**  
**Examination 2021 under cluster 9 (FAMT)**

Examinations Commencing from 1<sup>st</sup> June 2021

Program: **BE(MECHANICAL)**

Curriculum Scheme: Rev2019 'C' Scheme

Examination: SE Semester IV

Course Code: **MEC401** and Course Name: **Engineering Mathematics-4**

Time: 2 hour

Max. Marks: 80

Q1.	Choose the correct option for following questions. All the Questions are compulsory and carry equal marks
1.	Find the value of a if $\vec{F} = (x - 2z)i + (y - 5x)j + (az + 2x)k$ is solenoidal
Option A:	$a = 2$
Option B:	$a = -2$
Option C:	$a = -4$
Option D:	$a = 4$
2.	Vector field is Irrotational if
Option A:	$\nabla \times \vec{f} = 0$
Option B:	$\nabla \cdot \vec{f} = 0$
Option C:	$\nabla \times \vec{f} \neq 0$
Option D:	$\nabla \cdot \vec{f} = 1$
3.	The residue at the pole $z = -1$ of $f(z) = \frac{1}{(z+1)(z-2)^2}$ is
Option A:	1/3
Option B:	-1/3
Option C:	1/9
Option D:	-1/9
4.	The poles of $f(z) = \frac{3z-1}{(z+1)(z-2)}$ are
Option A:	1, -2
Option B:	-1, -2
Option C:	-1, 2
Option D:	1, 2
5.	Value of $\int_C \frac{\sin 2z \, dz}{(z + \pi/3)^4}$ is where $C:  z  = 2$
Option A:	$4\pi i/3$
Option B:	$\pi i/3$
Option C:	$2\pi i/3$
Option D:	$4\pi i$
6.	The value of $\int_0^{1+i} \bar{z} \, dz$ along straight line $y=x$ is
Option A:	0
Option B:	2

Option C:	3										
Option D:	1										
7.	If the two regression coefficient are -8/15 and -5/6 then the correlation coefficient is										
Option A:	0.667										
Option B:	-0.507										
Option C:	-0.667										
Option D:	0.607										
8.	Line of regression y on x is $8x-10y +66 =0$ . Line of regression x on y is $40x -18y -214 =0$ . The value of variance of y is 16. The standard deviation of x is										
Option A:	3										
Option B:	2										
Option C:	6										
Option D:	7										
9.	$\sum xy = 2638, \bar{x} = 14, \bar{y} =17, n=10$ then cov (x,y) is										
Option A:	24.2										
Option B:	25.8										
Option C:	23.9										
Option D:	20.5										
10.	Least square fit for the straight line $y= ax + b$ to the data <table border="1"><tr><td>x</td><td>1</td><td>2</td><td>3</td></tr><tr><td>y</td><td>5</td><td>7</td><td>9</td></tr></table>	x	1	2	3	y	5	7	9		
x	1	2	3								
y	5	7	9								
Option A:	$y = 2x +4$										
Option B:	$y = 2x -3$										
Option C:	$y = 2x +3$										
Option D:	$y = 3x -4$										
11.	A random variable X has the following probability distribution. The value of K is <table border="1"><tr><td>x</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>P(x)</td><td>5/K</td><td>7/K</td><td>9/K</td><td>11/K</td></tr></table>	x	2	3	4	5	P(x)	5/K	7/K	9/K	11/K
x	2	3	4	5							
P(x)	5/K	7/K	9/K	11/K							
Option A:	16										
Option B:	8										
Option C:	48										
Option D:	32										
12.	In Poisson distribution if $n = 100, p = 0.01$ , then the value of $P(r = 0)$										
Option A:	$1/e$										
Option B:	$2/e$										
Option C:	$3/e$										
Option D:	$1/4e$										
13.	A continuous random variable X has pdf $f(x) = kx; 0 \leq x \leq 1$ and $k; 1 \leq x \leq 2$ .then the value of k										
Option A:	2										
Option B:	2/3										

Option C:	3/2
Option D:	3
14.	If random variable X takes the values of $x=1,2,3$ with corresponding Probabilities $1/6, 2/3, 1/6$ then $E(x)$ is
Option A:	1
Option B:	3
Option C:	4
Option D:	2
15.	Number of road accident on a highway during a month follows a Poisson distribution with mean 2. Probability that in certain month number of accidents in the highway will be equal to 2 is
Option A:	0.354
Option B:	0.2707
Option C:	0.435
Option D:	0.521
16.	In a normal distribution when mean is 1 and S.D =3 then for the intervals $-1.43 \leq x \leq 6.19$ (for $z = -0.81$ , $A = 0.2910$ , for $z = 1.73$ , $A = 0.4582$ )
Option A:	0.7492
Option B:	0.4582
Option C:	0.2910
Option D:	0.1672
17.	X is normally distributed $\mu = 15$ , $\sigma^2 = 9$ . Given that for $z=1$ , $A=0.3413$ $P(X \geq 18)$ is given by
Option A:	0.1587
Option B:	0.4231
Option C:	0.2231
Option D:	0.3413
18.	In normal distribution. The area under standard normal curve to the right of y axis is
Option A:	1
Option B:	0
Option C:	0.5
Option D:	0.6
19.	If observed frequencies are 5,10,15 and expected frequencies are each equal to 10 then chi square value is
Option A:	20
Option B:	10
Option C:	15
Option D:	5
20.	Among 64 offspring of a certain cross between guinea pig 34 were red,10 were black and 20 were white, According to genetic model these number should in the ratio 9:3:4. Expected frequencies in the order
Option A:	36,12,16
Option B:	12,36,16
Option C:	20,12,16

Option D:	36,12,35
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Q2	Solve any Four out of Six	5 marks each																
A	<i>Evaluate by Green's theorem for the field</i> $\vec{F} = x^2\hat{i} + xy\hat{j}$ over the region R enclosed by $y=x^2$ and line $y=x$																	
B	Evaluate $\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$ ; c is $ z  = 3$																	
C	Determine the coefficient of correlation between X & Y from the following data <table><tr><td>X</td><td>51</td><td>54</td><td>56</td><td>59</td><td>65</td><td>60</td><td>70</td></tr><tr><td>Y</td><td>38</td><td>44</td><td>33</td><td>36</td><td>33</td><td>23</td><td>13</td></tr></table>		X	51	54	56	59	65	60	70	Y	38	44	33	36	33	23	13
X	51	54	56	59	65	60	70											
Y	38	44	33	36	33	23	13											
D	There is working women's hostel in a town where 75 % are from neighboring town, the rest all are from same town. 48% of women who hail from same town are graduates and 83 % of the women who have come from neighboring town are also graduates. Find the probability that a woman selected at a random is graduates from the same town.																	
E	In a certain examination test 2000 students appeared in a subject of statistics. Average marks obtained were 50% with standard deviation 5%. How many students do you expect to obtain more than 60% of marks, supposing that marks are distributed normally? (For $z = 2$ , $A = 0.4772$ )																	
F	The following table gives the number of accidents in a district during a week. Apply chi-square test to find whether the accidents are uniformly distributed over the week. <table><tr><td>Day</td><td>Sun</td><td>Mon</td><td>Tues</td><td>Wed</td><td>Thu</td><td>Fri</td><td>Sat</td></tr><tr><td>No.of accidents</td><td>13</td><td>12</td><td>11</td><td>9</td><td>15</td><td>10</td><td>14</td></tr></table> ( Table value of $\chi^2 = 12.59, d.f = 6, \text{level of significance} = 5\%$ )		Day	Sun	Mon	Tues	Wed	Thu	Fri	Sat	No.of accidents	13	12	11	9	15	10	14
Day	Sun	Mon	Tues	Wed	Thu	Fri	Sat											
No.of accidents	13	12	11	9	15	10	14											

Q3	Solve any Four out of Six	5 marks each																															
A	Evaluate using Stokes theorem $\iint_s (\nabla \times \vec{f}) \cdot \hat{n} \, ds$ where $s$ is curve surface of the paraboloid $x^2 + y^2 = 2z$ bounded by the plane $z=2$ where $\vec{f} = 3(x - y)\hat{i} + 2xz\hat{j} + xy\hat{k}$																																
B	Obtain Laurent's series expansions of $f(z) = \frac{z-1}{z^2-2z-3};  z  > 3$																																
C	Calculate the Spearman's rank correlation coefficient for the following data. <table><tr><td>x</td><td>32</td><td>55</td><td>49</td><td>60</td><td>43</td><td>37</td><td>43</td><td>49</td><td>10</td><td>20</td></tr><tr><td>y</td><td>40</td><td>30</td><td>70</td><td>20</td><td>30</td><td>50</td><td>72</td><td>60</td><td>45</td><td>25</td></tr></table>											x	32	55	49	60	43	37	43	49	10	20	y	40	30	70	20	30	50	72	60	45	25
x	32	55	49	60	43	37	43	49	10	20																							
y	40	30	70	20	30	50	72	60	45	25																							
D	A C.R.V X has the following pdf. $f(x) = k(x - x^2); 0 \leq x \leq 1$ Find K and mean																																
E	Ten individuals are chosen at random from a population & their height are found to be (inches): 63,63,64,65,66,69,69,70,70 & 71. In the light of the data, discuss the suggestion that the mean height in the population is 66 inches. (Table value of $t_{\alpha}=2.6$ , d.f =9, level of significance = 5%))																																
F	Standard deviation of two samples of size 9 & 13 were found to be 12.15 & 11.85. Can it be concluded that the samples were drawn from the normal population with the same standard deviation? (Given $F_{0.025} = 3.51$ for d. o. f. 8 & 12 & $F_{0.025} = 4.20$ for d. o. f. 12 & 8)																																