University of Mumbai

Examination 2021 under cluster __ (Lead College: _________)

Examinations Commencing from 1st June 2021 to 10th June 2021

Program: B.E.(Information Technology)

Curriculum Scheme: Rev-2019 'C' Scheme

Examination: S.E. Semester IV

Course Code: ITC 401 Course Name: Engineering Mathematics IV

Time: 2 hour Max. Marks: 80

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Q1.	**Choose the correct option for following questions. All the Questions are compulsory and carry equal marks**
1.	The region of rejection of the null hypothesis H_0 is known as
Option A:	Critical region
Option B:	Favourable region
Option C:	Domain
Option D:	Confidence region
2.	Sample of two types of electric bulbs were tested for length of life and the following data were obtained <table><tr><td></td><td>Size</td><td>Mean</td><td>SD</td></tr><tr><td>Sample 1</td><td>8</td><td>1234 h</td><td>36 h</td></tr><tr><td>Sample 2</td><td>7</td><td>1036 h</td><td>40 h</td></tr></table> The absolute value of test statistic in testing the significance of difference between means is
Option A:	t=10.77
Option B:	t=9.39
Option C:	t=8.5
Option D:	t=6.95
3.	If X is a poisson variate such that $P(X=1)=P(X=2)$, then $P(X=3)$ is
Option A:	$\frac{4e^2}{3}$
Option B:	$4e^2$
Option C:	$\frac{4}{3e^2}$
Option D:	$\frac{4}{e^2}$

4.	If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, Then following is not the eigenvalue ofadj A.
Option A:	6
Option B:	2
Option C:	4
Option D:	3
5.	For the matrix $\begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ the eigenvector corresponding to the distinct eigenvalue $\lambda = 2$ is
Option A:	$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$
Option B:	$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$
Option C:	$\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$
Option D:	$\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$
6.	The necessary and sufficient condition for a square matrix to be diagonalizable is that for each of it's eigenvalue
Option A:	algebraic multiplicity > geometric multiplicity
Option B:	algebraic multiplicity = geometric multiplicity
Option C:	algebraic multiplicity < geometric multiplicity
Option D:	algebraic multiplicity ≠ geometric multiplicity
7.	If the characteristic equation of a matrix A of order 3×3 is $\lambda^3 - 7\lambda^2 + 11\lambda - 5 = 0$, then by the Cayley-Hamilton theorem A^{-1} is equal to
Option A:	$\frac{1}{5}(A^3 - 7A^2 + 11\text{A})$
Option B:	$\frac{1}{5}(A^2 + 7A + 11\text{I})$
Option C:	$\frac{1}{5}(A^3 + 7A^2 + 11\text{A})$
Option D:	$\frac{1}{5}(A^2 - 7A + 11\text{I})$
8.	Value of an integral $\int_0^{1+i}(x^2 - iy)dz$ along the path $y = x^2$ is
Option A:	$\frac{5}{6} - \frac{i}{6}$
Option B:	$-\frac{5}{6} - \frac{i}{6}$
Option C:	$\frac{5}{6} + \frac{i}{6}$
Option D:	$\frac{-5}{6} + \frac{i}{6}$

9.	Integral $\int \frac{5z^2+7z+1}{z+1}\, dz$ along a circle $\lvert z \rvert = \frac{1}{2}$ is equal to
Option A:	1
Option B:	-1
Option C:	3/2
Option D:	0
10.	Analytic function gets expanded as a Laurent series if the region of convergence is
Option A:	Rectangular
Option B:	Triangular
Option C:	Circular
Option D:	Annular
11.	Residue of $f(z) = \frac{z^2}{(z+1)^2(z-2)}$ at a pole $z = 2$ is
Option A:	4/9
Option B:	2/9
Option C:	1/2
Option D:	0
12.	z-transform of an unit impulse function $\delta(k) = \begin{matrix} 1\,, & at\; k = 0 \\ 0\,, & otherwise \end{matrix}$ is
Option A:	1
Option B:	0
Option C:	-1
Option D:	K
13.	$z\{\sin(3k+5)\},\ k \geq 0$ is
Option A:	$\frac{z^2 \sin 2 - z \sin 5}{z^2 - 2zcos\, 3 + 1}$
Option B:	$\frac{z^2 \sin 5 + z \sin 2}{z^2 - 2zcos\, 3 + 1}$
Option C:	$\frac{z^2 \sin 5 - z \sin 2}{z^2 - 2zcos\, 3 + 1}$
Option D:	$\frac{z^2 \sin 2 + z \sin 5}{z^2 - 2zcos\, 3 + 1}$
14.	The inverse z-transform of $f(z) = \frac{z}{(z-1)(z-2)}$, $\lvert z \rvert > 2$ is
Option A:	$2^k - 2$
Option B:	$2^k - 1$
Option C:	$2^k + 1$
Option D:	$2^k + 2$
15.	If the basic solution of LPP is $x = 1, y = 0$ then the solution is
Option A:	Feasible and non-Degenerate
Option B:	Non-Feasible and Degenerate
Option C:	Feasible and Degenerate
Option D:	Non-Feasible and non-Degenerate

16.	If the primal LPP has an unbounded solution then the dual has
Option A:	Unbounded solution
Option B:	Bounded solution
Option C:	Feasible solution
Option D:	Infeasible solution
17.	Dual of the following LPP is Maximize $z = 2x_1 + 9x_2 + 11x_3$ $x_1 - x_2 + x_3 \geq 3$ Subject to $-3x_1 + 2x_3 \leq 1$ $2x_1 + x_2 - 5x_3 = 1$ $x_1, x_2, x_3 \geq 0$
Option A:	Minimize $w = -3y_1 + y_2 + y'$ $-y_1 - 3y_2 + 2y' \geq 2$ Subject to $y_1 + y' \geq 9$ $-y_1 + 2y_2 - 5y' \geq 11$ $y_1, y_2 \geq 0$, y' unrestricted
Option B:	Minimize $w = -3y_1 + y_2 + y_3$ $-y_1 - 3y_2 + 2y_3 \geq 2$ Subject to $y_1 + y_3 \geq 9$ $-y_1 + 2y_2 - 5y_3 \geq 11$ $y_1, y_2, y_3 \geq 0$
Option C:	Minimize $w = 2y_1 + 9y_2 + 11y'$ $-y_1 - 3y_2 + 2y' \geq 3$ Subject to $y_1 + y' \geq 1$ $-y_1 + 2y_2 - 5y' \geq 1$ $y_1, y_2 \geq 0$, y' unrestricted
Option D:	Minimize $w = 2y_1 + 9y_2 + 11y_3$ $-y_1 - 3y_2 + 2y_3 \geq 3$ Subject to $y_1 + y_3 \geq 1$ $-y_1 + 2y_2 - 5y_3 \geq 1$ $y_1, y_2 \geq 0$, y' unrestricted
18.	Consider the NLPP: Maximize $z = f(x_1, x_2)$, subject to the constraint $h = g(x_1, x_2) - b \leq 0$. Let $L = f - \lambda g$, then the Kuhn-Tucker conditions are
Option A:	$\frac{\partial L}{\partial x_1} \geq 0, \quad \frac{\partial L}{\partial x_2} \geq 0, \quad \lambda h \geq 0, \quad h \geq 0, \quad \lambda \geq 0$
Option B:	$\frac{\partial L}{\partial x_1} = 0, \quad \frac{\partial L}{\partial x_2} = 0, \quad \lambda h = 0, \quad h \leq 0, \quad \lambda \geq 0$
Option C:	$\frac{\partial L}{\partial x_1} = 0, \quad \frac{\partial L}{\partial x_2} = 0, \quad \lambda h \geq 0, \quad h \leq 0, \quad \lambda \leq 0$
Option D:	$\frac{\partial L}{\partial x_1} \geq 0, \quad \frac{\partial L}{\partial x_2} \geq 0, \quad \lambda h \geq 0, \quad h \geq 0, \quad \lambda = 0$
19.	In a non-linear programming problem,
Option A:	All the constraints should be linear
Option B:	All the constraints should be non-linear

Option C:	Either the objective function or atleast one of the constraints should be non-linear
Option D:	The objective function and all constraints should be linear.
20.	Pick the non-linear constraint
Option A:	$xy + y \geq 7$
Option B:	$2x - y \leq 5$
Option C:	$x + y \leq 6$
Option D:	$x + 2y = 9$

Subjective/descriptive questions

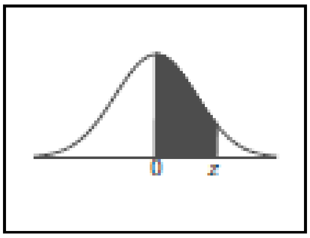
Q2 **(20 Marks)**	**Solve any Four out of Six5 marks each**
A	In an exam taken by 800 candidates, the average and standard deviation of marks obtained (normally distributed) are 40% and 10% respectively. What should be the minimum score if 350 candidates are to be declared as passed
B	If A= $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$, By using Cayley-Hamilton theorem find the matrix represented by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 + 2A + I$
C	Evaluate the following integral using Cauchy-Residue theorem. $I = \int_C \frac{z^2+3z}{\left(z+\frac{1}{4}\right)^2 (z-2)} dz$ where c is the circle $\left\lvert z - \frac{1}{2} \right\rvert = 1$
D	Obtain inverse z-transform $\frac{z+2}{z^2-2z-3}$, $1 < \lvert z \rvert < 3$
E	Solve by the Simplex method Maximize $z = 10x_1 + x_2 + x_3$ Subject to $x_1 + x_2 - 3x_3 \leq 10$ $4x_1 + x_2 + x_3 \leq 20$ $x_1, x_2, x_3 \geq 0$
F	Using Lagrange's multipliers solve the following NLPP Optimise $z = 4x_1 + 8x_2 - x_1^2 - x_2^2$ Subject to $x_1 + x_2 = 2$ $x_1, x_2 \geq 0$

Q3 **(20 Marks)**	**Solve any Four out of Six5 marks each**
A	When the first proof of 392 pages of a book of 1200 pages were read, the distribution of printing mistakes were found to be as follows. No of mistakes in page (X): 0, 1, 2, 3, 4 No. of pages (f): 275, 72, 30, 7, 5 Fit a poisson distribution to the above data and test the goodness of fit.

No of mistakes in page (X)	0	1	2	3	4
No. of pages (f)	275	72	30	7	5

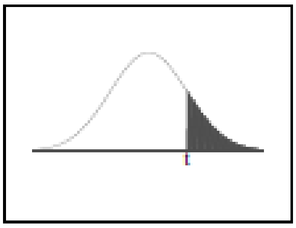
B	Show that the matrix $\begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$ is not diagonalizable.
C	If $f(z) = \frac{z-1}{(z-3)(z+1)}$ obtain Taylor's and Laurent's series expansions of f(z) in the domain $\lvert z\rvert < 1 \& 1 < \lvert z\rvert < 3$ respectively.
D	If $f(k) = \frac{1}{2^k} * \frac{1}{3^k}$ find $z\{f(k)\},\ k \geq 0$
E	Solve using dual simplex method Minimize $z = 2x_1 + 2x_2 + 4x_3$ $2x_1 + 3x_2 + 5x_3 \geq 2$ Subject to $3x_1 + x_2 + 7x_3 \leq 3$ $x_1 + 4x_2 + 6x_3 \leq 5$ $x_1, x_2, x_3 \geq 0$
F	Solve following NLPP using Kuhn-Tucker method Maximize $z = 2x_1^2 - 7x_2^2 - 16x_1 + 2x_2 + 12x_1x_2 + 7$ Subject to $2x_1 + 5x_2 \leq 105$ $x_1, x_2 \geq 0$

Standard Normal Distribution Table



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

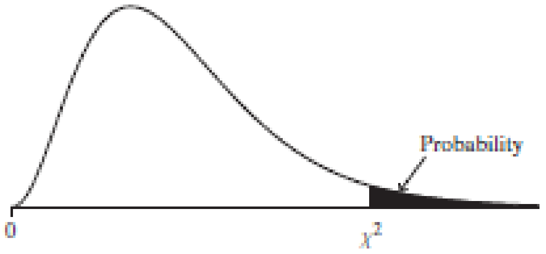
t-Distribution Table



The shaded area is equal to α for $t = t_\alpha$.

df	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
32	1.309	1.694	2.037	2.449	2.738
34	1.307	1.691	2.032	2.441	2.728
36	1.306	1.688	2.028	2.434	2.719
38	1.304	1.686	2.024	2.429	2.712
∞	1.282	1.645	1.960	2.326	2.576

TABLE C: Chi-Squared Distribution Values for Various Right-Tail Probabilities



	Right-Tail Probability						
df	0.250	0.100	0.050	0.025	0.010	0.005	0.001
1	1.32	2.71	3.84	5.02	6.63	7.88	10.83
2	2.77	4.61	5.99	7.38	9.21	10.60	13.82
3	4.11	6.25	7.81	9.35	11.34	12.84	16.27
4	5.39	7.78	9.49	11.14	13.28	14.86	18.47
5	6.63	9.24	11.07	12.83	15.09	16.75	20.52
6	7.84	10.64	12.59	14.45	16.81	18.55	22.46
7	9.04	12.02	14.07	16.01	18.48	20.28	24.32
8	10.22	13.36	15.51	17.53	20.09	21.96	26.12
9	11.39	14.68	16.92	19.02	21.67	23.59	27.88
10	12.55	15.99	18.31	20.48	23.21	25.19	29.59
11	13.70	17.28	19.68	21.92	24.72	26.76	31.26
12	14.85	18.55	21.03	23.34	26.22	28.30	32.91
13	15.98	19.81	22.36	24.74	27.69	29.82	34.53
14	17.12	21.06	23.68	26.12	29.14	31.32	36.12
15	18.25	22.31	25.00	27.49	30.58	32.80	37.70
16	19.37	23.54	26.30	28.85	32.00	34.27	39.25
17	20.49	24.77	27.59	30.19	33.41	35.72	40.79
18	21.60	25.99	28.87	31.53	34.81	37.16	42.31
19	22.72	27.20	30.14	32.85	36.19	38.58	43.82
20	23.83	28.41	31.41	34.17	37.57	40.00	45.32
25	29.34	34.38	37.65	40.65	44.31	46.93	52.62
30	34.80	40.26	43.77	46.98	50.89	53.67	59.70
40	45.62	51.80	55.76	59.34	63.69	66.77	73.40
50	56.33	63.17	67.50	71.42	76.15	79.49	86.66
60	66.98	74.40	79.08	83.30	88.38	91.95	99.61
70	77.58	85.53	90.53	95.02	100.4	104.2	112.3
80	88.13	96.58	101.8	106.6	112.3	116.3	124.8
90	98.65	107.6	113.1	118.1	124.1	128.3	137.2
100	109.1	118.5	124.3	129.6	135.8	140.2	149.5