University of Mumbai

Examination 2021 under cluster __ (Lead College: _____)

Examinations Commencing from 1st June 2021 to 10th June 2021

Program: **BE Electronics and Telecommunication Engineering**

Curriculum Scheme: Rev 2019 'C' Scheme

Examination: SE Semester IV

Course Code: **ECC401** and Course Name: **Engineering Mathematics IV**

Time: 2 hour Max. Marks: 80

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Note : Q1 carrying 40 marks. Q2 and Q3 are carrying 20 equal marks.

Q1.	**Choose the correct option for following questions. All the Questions are compulsory and carry equal marks. 2 marks each**
1.	If x is a discrete random variable with the following probability distribution x | 1 | 2 | 3 P(x) | a | 2a | a Find P(X≤ 2).
Option A:	$\frac{1}{4}$
Option B:	$\frac{1}{2}$
Option C:	$\frac{3}{4}$
Option D:	1
2.	Find E(X) if X has the p.d.f f(x) = $\begin{cases} \frac{3}{4}(2x - x^2) & ,0 \le x \le 2 \\ 0 & ,otherwise \end{cases}$
Option A:	$\frac{3}{2}$
Option B:	1
Option C:	2
Option D:	$\frac{1}{2}$
3.	If X and Y are independent random variables with means 2,3 and variance 1,2 respectively, find the mean and variance of the random variable Z = 2X - 5Y
Option A:	-11 , 54
Option B:	19 , 54
Option C:	19 , -8
Option D:	-11, -8
4.	Suppose the number of accidents occurring weekly on a particular stretch of a highway follow a Poisson distribution with mean **3** .Calculate the probability that there is at least one accident this week.
Option A:	0.6 347
Option B:	0.9502

Option C:	0.7275
Option D:	0.8002
5.	The following results were obtained from records of age (x) and systolic blood pressure (y) of a group of 10 men: <table><tr><td></td><td>x</td><td>y</td></tr><tr><td>mean</td><td>53</td><td>142</td></tr><tr><td>variance</td><td>130</td><td>165</td></tr></table> Correlation coefficient = 0.8 Estimate the blood pressure of a man whose age is 45?
Option A:	134.78
Option B:	130.56
Option C:	129.56
Option D:	137.56
6.	A coefficient of correlation is computed to be -0.95 means that
Option A:	The relationship between the two variables is weak
Option B:	The relationship between the two variables is strong and positive.
Option C:	The relationship between the two variables is strong but negative.
Option D:	The correlation coefficient cannot have this value.
7.	If the tangent of the angle made by the line of regression of y on x is 0.6 and $\sigma_x = \frac{1}{2}\sigma_y$ Find the correlation coefficient between x and y.
Option A:	$-$ 2 .5
Option B:	0. 25
Option C:	$-$ 0. 3
Option D:	0. 3
8.	Evaluate $\int_c \frac{7z-1}{(z-3)(z+5)} dz$, where c is the circle $\|z\|$ =1.
Option A:	$2\pi i$
Option B:	0
Option C:	6 πi
Option D:	πi
9.	Find the residue of $f(z) = \frac{z^2}{(z+2)(z-1)^2}$ at z = -2
Option A:	1/9
Option B:	5/9
Option C:	1/3
Option D:	4/9
10.	Identify the type of singularity of the function $f(z) = \frac{sinhz}{z^7}$
Option A:	z = 0 is a pole of order 7 for the given function
Option B:	z = 0 is a pole of order 6 for the given function
Option C:	z = 0 is an essential singularity

Option D:	z = 0 is a pole of order 3 for the given function
11.	Evaluate $\int_C \frac{e^z}{z-1} dz$ where C where c is the circle $\|z\|$ =2.
Option A:	$2\pi i$
Option B:	$2\pi i e^2$
Option C:	$2\pi i$e
Option D:	$\pi i e^2$
12.	Find the value of the integral $\int_0^{1+i}(x^2 - iy)dz$ along the path y = x
Option A:	$\frac{5-i}{6}$
Option B:	$\frac{5+i}{6}$
Option C:	$\frac{1+5i}{6}$
Option D:	$\frac{1-5i}{6}$
13.	Find the vector orthogonal to (2,1,−2) and (1,2,2)
Option A:	(1,−2, 1)
Option B:	(2, −2, 1)
Option C:	(1, −1, 1)
Option D:	(2 , 2, −1)
14.	If u = (3, 1, 4, −2) v = (2 , 2, 0,1) then find $\langle u, v\rangle$ and $\|\|u\|\|$,$\|\|v\|\|$
Option A:	-6 , $\sqrt{30}$, $\sqrt{10}$
Option B:	5 , $\sqrt{2}$, $\sqrt{6}$
Option C:	5 , $\sqrt{30}$, 3
Option D:	6 , $\sqrt{30}$, 3
15	Determine which of the following are subspaces of R^3 W_1={$(a, 0, b), a, b \in R$} W_2={$(a, b, 1), a, b \in R$}
Option A:	$W_1 and\ W_2$ are the subspaces of R^3
Option B:	W_1 and $W_2\ are$ not the subspaces of R^3
Option C:	$W_1 is\ a\ subapace\ of\ R^3$ but $W_2\ is$ not a subspace of R^3
Option D:	$W_1 is\ not\ a\ subapace\ of\ R^3$ but $W_2\ is$ a subspace of R^3
16.	Write down the matrix of the quadratic form $x_1{}^2 + 2x_2{}^2 - 7x_3{}^2 - 4x_1x_2 + 6x_2x_3 + 8x_3x_1$
Option A:	$\begin{bmatrix} 1 & -2 & 4 \\ -2 & 2 & 3 \\ 4 & 3 & -7 \end{bmatrix}$
Option B:	$\begin{bmatrix} 1 & -4 & 8 \\ -4 & 2 & 6 \\ 8 & 6 & -7 \end{bmatrix}$

Option C:	$\begin{bmatrix} 1 & 2 & 4 \\ 2 & 2 & 3 \\ 4 & 3 & -7 \end{bmatrix}$
Option D:	$\begin{bmatrix} 1 & 4 & 8 \\ 4 & 2 & 6 \\ 8 & 6 & 7 \end{bmatrix}$
17.	Find the rank , signature, index of the transformed quadratic form $3{y_1}^2 + \frac{2}{3}{y_2}^2 - \frac{39}{2}{y_3}^2$.
Option A:	rank = 3, signature =2, index =1
Option B:	rank = 3, signature =1, index =2.
Option C:	rank = 2, signature =3, index =1.
Option D:	rank = 2, signatur e=1, index =3.
18.	A necessary condition for I = $\int_{x_1}^{x_2} f(x, y, y^{\mid}, y^{\mid\mid})dx$ to be an extremal is that
Option A:	$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y^{\mid}}\right) + \frac{d^2}{dx^2}\left(\frac{\partial f}{\partial y^{\mid\mid}}\right) = 0$
Option B:	$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y^{\mid}}\right) = 0$
Option C:	$\frac{\partial f}{\partial y} + \frac{d}{dx}\left(\frac{\partial f}{\partial y^{\mid}}\right) = 0$
Option D:	$\frac{\partial f}{\partial y} + \frac{d}{dx}\left(\frac{\partial f}{\partial y^{\mid}}\right) + \frac{d^2}{dx^2}\left(\frac{\partial f}{\partial y^{\mid\mid}}\right) = 0$
19.	The functional I=$\int_a^b (y^{\mid^2} + 12xy)dx$ has the following extremal with c_1 and c_2 as arbitrary constants.
Option A:	$c_1x^3 + c_2x$
Option B:	$x^2 + c_1x + c_2$
Option C:	$c_1x + c_2$
Option D:	$x^3 + c_1x + c_2$
20.	The extremal of the functional I=$\int_a^b (16y^2 - y^{\mid\mid^2} + x^2)dx$ is
Option A:	y = $c_1 cos2x + c_2 sin2x$
Option B:	y = $c_1e^{2x} + c_2e^{-2x}$
Option C:	y = $c_1e^{2x} + c_2e^{-2x} + c_3 cos2x + c_4 sin2x$
Option D:	y = $c_1e^{x} + c_2e^{-x} + c_3 cosx + c_4 sinx$

Q2. (20 Marks)	**Solve any Four out of Six.** **5 marks each**
A	Fit a Poisson distribution for the following distribution . x: 0, 1, 2, 3, 4, Total f: 43, 40, 25, 10, 2, 120
B	Obtain the rank correlation coefficient for the following data X: 68, 64, 75, 50, 64, 80, 75, 40, 55, 64 Y: 62, 58, 68, 45, 81, 60, 68, 48, 50, 70
C	Obtain two distinct Laurent's series of $f(z) = \frac{2z-3}{z^2-4z+3}$ about z = 4 indicating the region of convergence
D	Construct an orthonormal basis of R^3 $using$ Gram-Schmidt process to S = $\{(1,0,0), (3,7,-2), (0,4,1)\}$
E	Reduce the symmetric matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ to the diagonal form using congruent transformation and interpret the result in terms of quadratic forms
F	Find the curve on which the functional $\int_a^b \sqrt{1+y'^2}\,dx$ is extremum.

Q3. (20 Marks)	**Solve any Four out of Six.** **5 marks each**
A	In a sample of 1000 cases, the mean of a certain test is 14 and standard deviation is 2.5 Assuming the distribution to be normal ,find (i)how many students score between 12 and 15 ? (ii) how many score above 18? (iii) how many score below 8?
B	In a partially destroyed laboratory, record of an analysis of correlation data, the following results only are legible: σ_x =3. Regression equations: 8X−10Y = −66, 40X−18Y=214. What are: (i) the mean values X and Y, (ii) the correlation coefficient between X and Y, (iii) the standard deviation of Y
C	Evaluate $\oint_C \frac{sin\pi z^2 + cos\pi z^2}{(z-2)(z-3)} dz$ where C is the circle $\|z\|$ =4..
D	Let V be a set of positive real numbers with addition and scalar multiplication defined as $x + y = xy$ and $cx = x^c$ .Show that Vis a vector space under this addition and scalar multiplication.
E	Reduce the following quadratic form into canonical form. Q: $x_1{}^2 + 2x_2{}^2 + 3x_3{}^2 - 2x_1x_3 + 2x_2x_3 + 2x_2x_1$
F	Using Rayleigh -Ritz method , solve the boundary value problem I=$\int_0^1 (y'^2 - y^2 - 2xy)dx$ with y(0)=0 and y(1)=0.