

**University of Mumbai**  
**Examination 2021 under cluster \_\_ (Lead College: \_\_\_\_\_)**

**Examinations Commencing from 1<sup>st</sup> June 2021 to 10<sup>th</sup> June 2021**

**Program: BE(Instrumentation Engineering)**

Curriculum Scheme: Rev2016

Examination: SE Semester IV

Course Code: ISC401 and Course Name: Applied Mathematics-IV

Time: 2-hour

Max. Marks: 80

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| Q1.       | Choose the correct option for following questions. All the Questions are compulsory and carry equal marks |
|-----------|-----------------------------------------------------------------------------------------------------------|
| 1.        | What is the value of $\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta} d\theta$                           |
| Option A: | $\frac{\pi}{4}$                                                                                           |
| Option B: | $\frac{\pi}{5}$                                                                                           |
| Option C: | $\frac{\pi}{6}$                                                                                           |
| Option D: | $\frac{\pi}{7}$                                                                                           |
| 2.        | What is the value of $\int_c \frac{2(3z^2+z)}{(z-1)(z+1)} dz$ where $c$ is circle $ z  = 2$               |
| Option A: | $2\pi i$                                                                                                  |
| Option B: | $4\pi i$                                                                                                  |
| Option C: | $\pi i$                                                                                                   |
| Option D: | $\pi$                                                                                                     |
| 3.        | If $x, y$ are independent variables then they are _____                                                   |
| Option A: | Related                                                                                                   |
| Option B: | Correlated                                                                                                |
| Option C: | Not correlated                                                                                            |
| Option D: | Semi related                                                                                              |
| 4.        | What is the value of $\int_c \frac{z+2}{(z-3)(z-4)} dz$ , where $c$ is circle $ z  = 1$ .                 |
| Option A: | 0                                                                                                         |
| Option B: | 1                                                                                                         |
| Option C: | 2                                                                                                         |
| Option D: | 3                                                                                                         |
| 5.        | The equation of the line of regression of $Y$ on $X$ is ____                                              |
| Option A: | $Y - \bar{Y} = b_{yx}(X - \bar{X})$                                                                       |
| Option B: | $Y - \bar{Y} = b_{yx}(X + \bar{X})$                                                                       |
| Option C: | $Y + \bar{Y} = b_{yx}(X - \bar{X})$                                                                       |
| Option D: | $Y - \bar{Y} = b_{xy}(X - \bar{X})$                                                                       |
| 6.        | Binomial Distribution is a _____                                                                          |
| Option A: | Continuous distribution                                                                                   |
| Option B: | Discrete distribution                                                                                     |

|           |                                                                                                                                                                                                      |
|-----------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Option C: | Irregular distribution                                                                                                                                                                               |
| Option D: | Not a Probability distribution                                                                                                                                                                       |
| 7.        | Total area under the curve of a continuous probability density function is always equal to:                                                                                                          |
| Option A: | 0                                                                                                                                                                                                    |
| Option B: | -1                                                                                                                                                                                                   |
| Option C: | 1                                                                                                                                                                                                    |
| Option D: | 2                                                                                                                                                                                                    |
| 8.        | If 'm' is the mean of Poisson Distribution, the P (0) is given by _____                                                                                                                              |
| Option A: | $e^{-m}$                                                                                                                                                                                             |
| Option B: | $e^m$                                                                                                                                                                                                |
| Option C: | E                                                                                                                                                                                                    |
| Option D: | $m^{-e}$                                                                                                                                                                                             |
| 9.        | If $f(z)$ is analytic in and on closed contour $C$ then $\oint_C f(z)dz = \underline{\hspace{2cm}}$                                                                                                  |
| Option A: | 1                                                                                                                                                                                                    |
| Option B: | -1                                                                                                                                                                                                   |
| Option C: | 2                                                                                                                                                                                                    |
| Option D: | 0                                                                                                                                                                                                    |
| 10.       | If $\lambda_1$ and $\lambda_2$ are eigen values of square matrix A of order two then $\det(A) = \underline{\hspace{2cm}}$                                                                            |
| Option A: | $\lambda_1 + \lambda_2$                                                                                                                                                                              |
| Option B: | $\lambda_1 \cdot \lambda_2$                                                                                                                                                                          |
| Option C: | $\lambda_1 - \lambda_2$                                                                                                                                                                              |
| Option D: | $\lambda_1 + 2 \lambda_2$                                                                                                                                                                            |
| 11.       | Let A be a $3 \times 3$ matrix with real entries such that $\det(A)=6$ and the trace of A is 0. If $\det(A+I) = 0$ , where I denote the $3 \times 3$ identity matrix, then the eigen values of A are |
| Option A: | -1,2,3                                                                                                                                                                                               |
| Option B: | -1, 2, -3                                                                                                                                                                                            |
| Option C: | 1, 2, -3                                                                                                                                                                                             |
| Option D: | -1, -2, 3                                                                                                                                                                                            |
| 12.       | In a Binomial Distribution, if 'n' is the number of trials and 'p' is the probability of success, then the mean value is given by                                                                    |
| Option A: | n.p                                                                                                                                                                                                  |
| Option B: | n.p.q                                                                                                                                                                                                |
| Option C: | $n.p^2q$                                                                                                                                                                                             |
| Option D: | $n.p.q^2$                                                                                                                                                                                            |
| 13.       | If $A = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$ then minimal polynomial $f(x)$ of A is _____                                                                                                   |
| Option A: | $x^2 - 3x - 2$                                                                                                                                                                                       |
| Option B: | $x^2 + 3x + 2$                                                                                                                                                                                       |
| Option C: | $x^2 - 3x + 2$                                                                                                                                                                                       |
| Option D: | $x^2 - 3x + 22$                                                                                                                                                                                      |
| 14.       | The probability function of a random variable is defined as:                                                                                                                                         |

|           |                                                                                                                                                                                                       |    |    |    |    |   |   |      |   |    |    |    |    |
|-----------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|----|----|----|---|---|------|---|----|----|----|----|
|           | <table><tr><td>x</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td></tr><tr><td>P(x)</td><td>k</td><td>2k</td><td>3k</td><td>4k</td><td>5k</td></tr></table> <p>Then what is the value of k</p> | x  | -2 | -1 | 0  | 1 | 2 | P(x) | k | 2k | 3k | 4k | 5k |
| x         | -2                                                                                                                                                                                                    | -1 | 0  | 1  | 2  |   |   |      |   |    |    |    |    |
| P(x)      | k                                                                                                                                                                                                     | 2k | 3k | 4k | 5k |   |   |      |   |    |    |    |    |
| Option A: | $\frac{1}{14}$                                                                                                                                                                                        |    |    |    |    |   |   |      |   |    |    |    |    |
| Option B: | $\frac{1}{15}$                                                                                                                                                                                        |    |    |    |    |   |   |      |   |    |    |    |    |
| Option C: | $\frac{1}{16}$                                                                                                                                                                                        |    |    |    |    |   |   |      |   |    |    |    |    |
| Option D: | $\frac{1}{17}$                                                                                                                                                                                        |    |    |    |    |   |   |      |   |    |    |    |    |
|           |                                                                                                                                                                                                       |    |    |    |    |   |   |      |   |    |    |    |    |
| 15.       | If X is a random variable with PDF $f(x)$ then $\int_{-\infty}^{\infty} xf(x)dx$ is equal to                                                                                                          |    |    |    |    |   |   |      |   |    |    |    |    |
| Option A: | Variance                                                                                                                                                                                              |    |    |    |    |   |   |      |   |    |    |    |    |
| Option B: | Standard deviation                                                                                                                                                                                    |    |    |    |    |   |   |      |   |    |    |    |    |
| Option C: | Mean                                                                                                                                                                                                  |    |    |    |    |   |   |      |   |    |    |    |    |
| Option D: | Mode                                                                                                                                                                                                  |    |    |    |    |   |   |      |   |    |    |    |    |
|           |                                                                                                                                                                                                       |    |    |    |    |   |   |      |   |    |    |    |    |
| 16.       | If $v = (1 \ 3 \ -1)$ then $\ v\  =?$                                                                                                                                                                 |    |    |    |    |   |   |      |   |    |    |    |    |
| Option A: | $\sqrt{12}$                                                                                                                                                                                           |    |    |    |    |   |   |      |   |    |    |    |    |
| Option B: | $\sqrt{11}$                                                                                                                                                                                           |    |    |    |    |   |   |      |   |    |    |    |    |
| Option C: | $\sqrt{13}$                                                                                                                                                                                           |    |    |    |    |   |   |      |   |    |    |    |    |
| Option D: | $\sqrt{14}$                                                                                                                                                                                           |    |    |    |    |   |   |      |   |    |    |    |    |
|           |                                                                                                                                                                                                       |    |    |    |    |   |   |      |   |    |    |    |    |
| 17.       | The determinant of $3 \times 3$ matrix $\begin{bmatrix} 100 & 200 & 300 \\ 0 & 2 & 400 \\ 0 & 0 & -1 \end{bmatrix}$ is ____                                                                           |    |    |    |    |   |   |      |   |    |    |    |    |
| Option A: | 100                                                                                                                                                                                                   |    |    |    |    |   |   |      |   |    |    |    |    |
| Option B: | 200                                                                                                                                                                                                   |    |    |    |    |   |   |      |   |    |    |    |    |
| Option C: | -100                                                                                                                                                                                                  |    |    |    |    |   |   |      |   |    |    |    |    |
| Option D: | -200                                                                                                                                                                                                  |    |    |    |    |   |   |      |   |    |    |    |    |
|           |                                                                                                                                                                                                       |    |    |    |    |   |   |      |   |    |    |    |    |
| 18.       | If $W_1$ and $W_2$ are subspaces of vector space V then which of the following is always subspace of V?                                                                                               |    |    |    |    |   |   |      |   |    |    |    |    |
| Option A: | $W_1 \cap W_2$                                                                                                                                                                                        |    |    |    |    |   |   |      |   |    |    |    |    |
| Option B: | $W_1 \cup W_2$                                                                                                                                                                                        |    |    |    |    |   |   |      |   |    |    |    |    |
| Option C: | $W_1 - W_2$                                                                                                                                                                                           |    |    |    |    |   |   |      |   |    |    |    |    |
| Option D: | $W_1^c$                                                                                                                                                                                               |    |    |    |    |   |   |      |   |    |    |    |    |
|           |                                                                                                                                                                                                       |    |    |    |    |   |   |      |   |    |    |    |    |
| 19.       | Which of the following is not possible in probability distribution?                                                                                                                                   |    |    |    |    |   |   |      |   |    |    |    |    |
| Option A: | $p(x) \geq 0$                                                                                                                                                                                         |    |    |    |    |   |   |      |   |    |    |    |    |
| Option B: | $\sum p(x) = 1$                                                                                                                                                                                       |    |    |    |    |   |   |      |   |    |    |    |    |
| Option C: | $\sum xp(x) = 2$                                                                                                                                                                                      |    |    |    |    |   |   |      |   |    |    |    |    |
| Option D: | $p(x) = -0.5$                                                                                                                                                                                         |    |    |    |    |   |   |      |   |    |    |    |    |
|           |                                                                                                                                                                                                       |    |    |    |    |   |   |      |   |    |    |    |    |
| 20.       | If $u, v, w \in V$ (vectorspace) and $u + w = v + w$ then $u =$ ____                                                                                                                                  |    |    |    |    |   |   |      |   |    |    |    |    |
| Option A: | $v$                                                                                                                                                                                                   |    |    |    |    |   |   |      |   |    |    |    |    |
| Option B: | $w$                                                                                                                                                                                                   |    |    |    |    |   |   |      |   |    |    |    |    |
| Option C: | 1                                                                                                                                                                                                     |    |    |    |    |   |   |      |   |    |    |    |    |
| Option D: | 0                                                                                                                                                                                                     |    |    |    |    |   |   |      |   |    |    |    |    |

| Q2 | Solve any Four out of Six                                                                                           | 5 marks each |    |    |       |         |        |        |    |    |    |
|----|---------------------------------------------------------------------------------------------------------------------|--------------|----|----|-------|---------|--------|--------|----|----|----|
| A  | Let $u$ and $v$ be orthogonal vectors. If $u + v$ and $u - v$ are orthogonal, show that $ u  =  v $ .               |              |    |    |       |         |        |        |    |    |    |
| B  | Determine whether the matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ is derogatory. |              |    |    |       |         |        |        |    |    |    |
| C  | A random variable $X$ has following probability function                                                            |              |    |    |       |         |        |        |    |    |    |
|    | X                                                                                                                   | 1            | 2  | 3  | 4     | 5       | 6      | 7      |    |    |    |
|    | P (X= x)                                                                                                            | k            | 2k | 3k | $k^2$ | $k^2+k$ | $2k^2$ | $4k^2$ |    |    |    |
|    | Find i) k    ii) P (X ≥ 2)    iii) P (X < 5)    iv) P (0 ≤ X ≤ 5)                                                   |              |    |    |       |         |        |        |    |    |    |
| D  | Find the Binomial Distribution if the mean is 4 and variance is 3.                                                  |              |    |    |       |         |        |        |    |    |    |
| E  | Evaluate $\int_C \frac{zdz}{(z-2)(z-1)^2}$ Where $C$ is the circle $ z - 2  = 0.5$                                  |              |    |    |       |         |        |        |    |    |    |
| F  | Calculate the correlation coefficient from the following data.                                                      |              |    |    |       |         |        |        |    |    |    |
|    | X:                                                                                                                  | 23           | 27 | 28 | 29    | 30      | 31     | 33     | 35 | 36 | 39 |
|    | Y:                                                                                                                  | 18           | 22 | 23 | 24    | 25      | 26     | 28     | 29 | 30 | 32 |

| Q3 | Solve any Four out of Six                                                                                                                                                                                                                                                        | 5 marks each |    |    |    |    |    |   |    |    |   |    |    |    |    |    |    |    |
|----|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------|----|----|----|----|----|---|----|----|---|----|----|----|----|----|----|----|
| A  | Apply Gram-Schmidt orthogonalization process to the following sequence of vectors in $R^3$ , $u = (1 \ 2 \ 0)$ , $v = (8 \ 1 \ -6)$ , $w = (0 \ 0 \ 1)$ .                                                                                                                        |              |    |    |    |    |    |   |    |    |   |    |    |    |    |    |    |    |
| B  | If $A = \begin{bmatrix} \pi & \pi/4 \\ 0 & \pi/2 \end{bmatrix}$ then find $\cos A$ .                                                                                                                                                                                             |              |    |    |    |    |    |   |    |    |   |    |    |    |    |    |    |    |
| C  | A continuous random variable X has the probability density function<br>$f(x) = \begin{cases} kx & ; \ 0 \leq x \leq 2 \\ 2x & ; \ 2 \leq x \leq 4 \\ 6k - kx & ; \ 4 \leq x \leq 6 \end{cases}$<br>Then find I) k    II) $P(1 \leq x \leq 3)$ III) mean.                         |              |    |    |    |    |    |   |    |    |   |    |    |    |    |    |    |    |
| D  | In a Poisson distribution $P(x=3)$ is $2/3$ times $P(x=4)$ . Find the mean and standard deviation.                                                                                                                                                                               |              |    |    |    |    |    |   |    |    |   |    |    |    |    |    |    |    |
| E  | Evaluate $\int_C \frac{z^2 dz}{(z-2)(z-1)^2}$ Where C is the circle $ z =2.5$                                                                                                                                                                                                    |              |    |    |    |    |    |   |    |    |   |    |    |    |    |    |    |    |
| F  | Find the equations of the lines of regression for the following data. <table><tr><td>x</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td></tr><tr><td>y</td><td>11</td><td>14</td><td>14</td><td>15</td><td>12</td><td>17</td><td>16</td></tr></table> |              | x  | 5  | 6  | 7  | 8  | 9 | 10 | 11 | y | 11 | 14 | 14 | 15 | 12 | 17 | 16 |
| x  | 5                                                                                                                                                                                                                                                                                | 6            | 7  | 8  | 9  | 10 | 11 |   |    |    |   |    |    |    |    |    |    |    |
| y  | 11                                                                                                                                                                                                                                                                               | 14           | 14 | 15 | 12 | 17 | 16 |   |    |    |   |    |    |    |    |    |    |    |