University of Mumbai

Civil Engineering Examination

Sub: Engineering Mathematics-III **Year/Sem: SE/III**

Marks:80 **Duration:2 Hours**

Q1. Attempt all the MCQS **(20*2=40 marks)**

1) $L(e^{at}) =$

(A) $\frac{1}{s-a}, s > a$ (B) $\frac{1}{s+a}, s > a$ (C) $\frac{1}{s^2-a^2}, \quad s > a$ (D) $\frac{1}{s^2+a^2}, s > a$

2) Find $L[2t^3 + cosh4t]$

(A) $\frac{12}{s^4} + \frac{s}{s^2+16}$ (B) $\frac{48}{s^4} + \frac{s}{s^2+16}$ (C) $\frac{12}{s^4} + \frac{4}{s^2+16}$ (D) $\frac{12}{s^4} + \frac{s}{s^2-16}$

3) $L\{sin^2 t\} =$

(A) $\frac{4}{s^3+4s}$ (B) $\frac{2}{s^3+4s}$ (C) $\frac{1}{s(s^2+4)}$ (D) $None\ of\ these$

4) Find $L^{-1}\left(\frac{2s}{s^4+4}\right)$

(A) $4cost.sinht$ (B) $2cost.cosht$ (C) $sin3t.sinht$ (D) $sint.sinht$

5) Find $L^{-1}\left(\frac{s+2}{s^2+4s+7}\right)$

(A) $e^{-t}.sin\sqrt{3}t$ (B) $e^{-3t}.cosh\sqrt{3}t$ (C) $e^{-2t}.cos\sqrt{3}t$ (D) $e^{-4t}.cos6t$

6) In the interval $(-L, L)$, the b_n co-efficient is

(A) $b_n = \frac{1}{L}\int_{-l}^{l} f(x)\sin\left(\frac{n\pi x}{L}\right) dx$ (B) $b_n = \frac{2}{\pi}\int_{-L}^{L} f(x)\sin(n\pi)\, dx$

(C) $b_n = \frac{1}{\pi}\int_{-L}^{L} f(x)\sin(n\pi)\, dx$ (D) π

7) $If\ f(x) = x^2\ in\ (0,2\pi), then\ a_0 =$

$(A)\frac{2\pi^2}{3}$ $(B)\frac{\pi^2}{3}$ $(C)\frac{4\pi^2}{3}$ $(D)\frac{8\pi^2}{3}$

8)Find a_n if the function $f(x) = x - x^3$

(A)1 (B)Infinite value (C)Zero (D)Cannot be found

9) What is the for Parseval's relation in Fourier series expansion?

(A) $\int_{-l}^{l}(f(x))^2\,dx = l\left[\frac{a_0{}^2}{2} + \sum_{n=1}^{\infty}\left(a_n{}^2 + b_n{}^2\right)\right]$

(B) $\int_{-l}^{l}(f(x))^2\,dx = l\left[\frac{a_0{}^2}{2} + \sum_{n=1}^{\infty}\left(a_n{}^2\right)\right]$

(C) $\int_{-l}^{l}(f(x))^2\,dx = \frac{l}{2}\left[\frac{a_0{}^2}{2} + \sum_{n=1}^{\infty}\left(a_n{}^2 + b_n{}^2\right)\right]$

(D) $l\int_{-l}^{l}(f(x))^2\,dx = \left[\frac{a_0{}^2}{2} + \sum_{n=1}^{\infty}\left(a_n{}^2 + b_n{}^2\right)\right]$

10) For Half Range Sine Series of xsinx in (0,π), a_n is_____________

$(A)\frac{\pi}{8\sqrt{2}}$ $(B)0$ $(C)\frac{\pi}{8}$ $(D)None\ of\ these$

11) $If\ f(z) = \frac{1}{2}\log(x^2 + y^2) + i\tan^{-1}\frac{ay}{x}\ \ is\ analytic\ then\ the\ value\ of\ a =$ ____

(A)0 (B)2 (C)1 (D)-1

12) If u and v are harmonic function then $f(z) = u + iv$ is

(A) Need not be analytic function (B)Analytic function

(C)Analytic function at z=0 (D)Analytic function at z=i

13) If the real part of an analytic function $f(z)$ is $x^2 - y^2 - y$, then the imaginary part is

(A)$2xy$ (B)$x^2 + 2xy$ (C)$2xy - y$ (D)$2xy + x$

14)If f(z)=u+iv is analytic then

$(A)\ u_x = v_{y,}\ u_y = -v_x$ $(B) u_x = -v_{y,}\ u_y = v_x$

$(C) u_x = -v_{y,}\ u_y = -v_x$ $(D) u_x = v_{y,}\ u_y = v_x$

15)The eigenvalues of the $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ are

(A)2,2,2 (B)2,1,1 (C)2,1,0 (D)1,1,0

16)If $A = \begin{bmatrix} 1 & 8 \\ 2 & 1 \end{bmatrix}$ Find $2A^3 - A^2 - 35A - 44I$

(A)$A - 4I$ (B)$A + I$ (C)$5A + 3I$ (D)$15A + 7I$

17)If $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ & B= $\begin{bmatrix} 2 & 0 \\ 1/2 & 2 \end{bmatrix}$ then

(A) A and B both are not diagonalisable

(B) A and B both are diagonalisable

(C) A is diagonalizable but B is not diagonalisable

(D) A isnot diagonalizable but B is diagonalisable

18)Consider the following statements
i)The eigenvalues of Hermitian matrix are real
ii)Eigenvalues of skew Hermitian matrix are either purely imaginary or zero
iii) Eigen values of unitary matrix are of unit modulus.
Then

(A) statement i , ii are correct and iii is not correct

(B) statement i , is correct and ii, iii are not correct

(C) Statement i,ii,iii are not correct

(D) Statement i,ii,iii are correct statements.

19) $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ is a

A)One dimensional heat flow equation
B)One dimensional wave equation
C)Two dimensional heat flow equation
D) None of these

20)In Crank-Nicholson formula the step sizes are related by

(A) $k = \frac{a}{2}h^2$ (B)$k = ah^2$ (C) $k = a\frac{h^2}{2}$ (D)$k = a^2 h$

Q2. Attempt any FOUR **(04 X 05 marks= 20 marks)**

1)$Solve\ L\left\{\frac{sint.sinht}{t}\right\}\ and\ hence\ find \int_0^\infty e^{-2t}\frac{sint.sinht}{t}dt$

2) Using Convolution theorem find $L^{-1}\left\{\frac{(s+3)^2}{(s^2+6s+5)^2}\right\}$

3) Find the Fourier Series Expansion of $f(x) = 4 - x^2, 0 < x < 2$ and hence deduce that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{6}$

4)Find the family of curves orthogonal to $3x^2y - y^3 = c$

5) Show that the matrix $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ is daigonalizable and find the modal matrix P and the diagonal matrix D.

Q3. Attempt any FOUR **(04 X 05 marks= 20 marks)**

1)Find Half Range Fourier Sine Series of $f(x) = x(\pi - x); 0 < x < \pi$

2)Show that the function $u = e^x(xcosy - ysiny)$ is Harmonic and hence find the Harmonic Conjugate of it

3) Solve $L\{t\sqrt{1 + sint}\}$ and hence find $\int_0^\infty e^{-t}\ t\sqrt{1 + sint}\,$dt

4)Find Inverse Laplace Transform of $\frac{s^3+2s}{(s+1)^2(s^2+1)}$

5) Using Crank- Nicholson Simplified formula solve

$\frac{\partial^2 u}{\partial x^2} - 16\frac{\partial u}{\partial t} = 0, 0 < x < 1, t > 0 , u(x,t) = 0 , u(0,t) = 0, u(1,t) = 200t$.

Compute u for one step in t direction taking $h = \frac{1}{4}$